Section 1.5: Parametric Equation of a Line in $\mathbb{R}^n$

- Given a vector $\vec{\mathbf{v}} \in \mathbb{R}^n$ and a point $\vec{\mathbf{x}}_0 \in \mathbb{R}^n$, the parametric equation of the line passing through $\vec{\mathbf{x}}_0$ parallel to $\vec{\mathbf{v}}$ is

$$L(t) = \vec{\mathbf{v}}t + \vec{\mathbf{x}}_0.$$

- Example: Find a parametric equation for the line in $\mathbb{R}^3$ containing the points $(1, 3, -7)$ and $(2, -5, 1)$.

  The slope vector is obtained by subtracting the coordinates of the points: $\vec{\mathbf{v}} = \langle 1, -8, 8 \rangle$. The parametric equation is therefore

$$L(t) = \langle 1, -8, 8 \rangle t + \langle 1, 3, -7 \rangle.$$

  This can also be written as

$$L(t) = \langle t + 1, -8t + 3, 8t - 7 \rangle,$$

  or as a system of parametric equations:

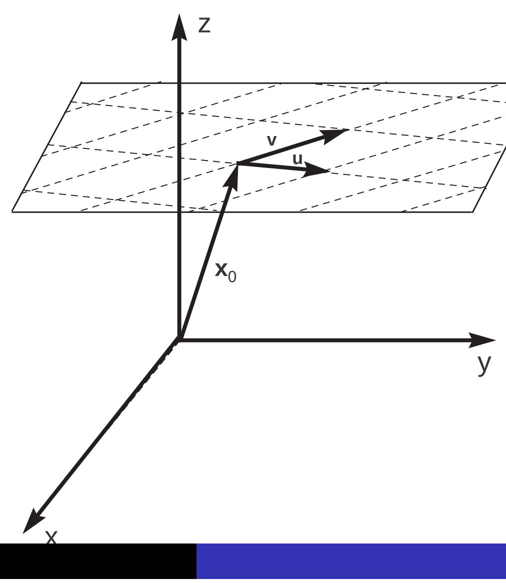
$$x = t + 1, \qquad y = -8t + 3, \qquad z = 8t - 7.$$

Parametric Equation of a Plane in $\mathbb{R}^3$

- Given two vectors $\vec{\mathbf{u}}, \vec{\mathbf{v}} \in \mathbb{R}^3$ such that neither $\vec{\mathbf{u}}$ nor $\vec{\mathbf{v}}$ is a scalar multiple of the other, the set

$$P(s, t) = \vec{\mathbf{u}}s + \vec{\mathbf{v}}t + \vec{\mathbf{x}}_0$$

  is a
  plane in $\mathbb{R}^3$ containing point $\vec{\mathbf{x}}_0$ and parallel to the vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$.



Equations Related to Normal Vectors

- Given a nonzero vector $\vec{\mathbf{n}}$ in $\mathbb{R}^2$ or $\mathbb{R}^3$, think about the set of all vectors emanating from a point $\vec{\mathbf{x}}_0$ that are perpendicular to $\vec{\mathbf{n}}$.
- The set of all such vectors forms a line through $\vec{\mathbf{x}}_0$ in $\mathbb{R}^2$, or a plane through $\vec{\mathbf{x}}_0$ in $\mathbb{R}^3$.
- The equation defining the set of all such vectors is

$$\vec{\mathbf{n}} \cdot (\vec{\mathbf{x}} - \vec{\mathbf{x}}_0) = 0.$$

- In $\mathbb{R}^2$, this shows that $Ax + By = C$ defines a line with normal vector $\langle A, B \rangle$.
- In $\mathbb{R}^3$, this shows that $Ax + By + Cz = D$ defines a plane with normal vector $\langle A, B, C \rangle$.